The VAT Versus The Turnover Tax With Non-Competitive Firms

Arindam Das-Gupta

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Abstract

The VAT is compared to a turnover tax (TT) given monopoly final goods and intermediate goods firms interacting strategically. Linear demands and constant costs are assumed. Via examples it is shown that for both "Cournot" and "Stackelberg" games, a revenue neutral VAT may not exist to a given turnover tax; and the TT can dominate the VAT simultaneously in welfare, revenue and output terms. In other examples it is shown that the VAT dominating the TT by all three indicators is also possible. It is also shown that outcomes are identical to the "Cournot" game when the consumer goods firm is the strategic leader. When the intermediate goods firm is the leader, intermediate price distortion is lower and welfare higher than in the "Cournot" game under both taxes; and the output neutral VAT rate to any feasible TT rate is higher than in the "Cournot" game.

Key words: VAT, Retail Sales Tax, Turnover Tax, Welfare, Tax Revenue, Cournot, Stackelberg

JEL Classification: D42, H21, H25

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Overview and Motivation

When the government wishes to raise a given amount of revenue and when its ability to levy taxes on different goods, except leisure, is not constrained, Diamond and Mirrlees (1971) show that a necessary condition for welfare maximising second best taxes on competitive firms is that there be no taxation of productive inputs. Dasgupta and Stiglitz (1972) extend this result to non-constant returns to scale. This is one of the major theoretical arguments favouring the consumption type Value Added Tax (VAT) over a general sales or turnover tax (TT). The former rebates tax paid by intermediate input producers and levies no tax on primary factors, while the latter taxes intermediate and possibly capital goods. Since apart from tax administration and information differences (not considered in these papers), a retail sales tax on consumer goods (RST) is equivalent to a consumption type VAT, the argument also applies to the RST.[1]

However, the result does not survive if the government's choice of fiscal instruments is constrained. This is shown, for example, by Stiglitz and Dasgupta (1971) when some sectors are not taxable or when taxing a subset of goods at different tax rates is infeasible.[2] Furthermore, Stiglitz and Dasgupta (1971) show that, if some producers are monopolists, it may be welfare maximising to sacrifice production

[1] *See*, for example, Ebrill, *et. al*. (2001).

[2] Other situations are where there is decreasing returns and limited profits taxation or where the government faces a budget constraint. For the case of non-taxable sectors, *see* also Newbery (1986).

efficiency and tax inputs. They also show that in the presence of monopolies or increasing returns in some sectors, the difference between price and marginal cost should be subsidised, with the subsidy being financed by distortionary input taxes if 100 percent taxes on profits are insufficient for budget balance. They do not examine whether input taxation is optimal when, given constraints, there is a tax rather than a subsidy on monopoly producers, as obtains in most real world tax systems.

Despite negative results concerning taxation of inputs, conjectured efficiency benefits and possible administrative advantages of the VAT[3] have led to it being introduced or replacing other taxes on production and sales, often the TT, in over 120 developed, developing and transition economies over the past 50 years.[4] A major exception, the United States, has state-level RSTs. Earlier analytical comparison of the VAT/RST with the TT and other taxes on production or sales have affirmed the superiority of the VAT in terms of welfare, tax revenue, output or productive efficiency.[5] These comparisons of the VAT/RST with the TT have either assumed a production technology with fixed intermediate input requirements per unit of output or competitive firms (or both).

The assumption of fixed proportions technology for final goods production is inappropriate when comparing a VAT to a TT, given the incentive for vertical integration with a TT, to avoid double taxation, recognised in the literature. Vertical integration clearly involves substitution of primary inputs for purchased intermediate inputs. The limited relevance of the assumption of competitive firms is also clear.

This paper presents the first negative results concerning the VAT compared to other sales taxes, to our knowledge. In the paper the VAT and the TT are compared when there is strategic interaction between non-competitive consumer goods and input producing firms. The framework assumes "text-book" linear consumer and intermediate good

[3] A theoretical examination of the advantages of transactions cross-matching, which is claimed to be a possibly important administrative advantage of the VAT is in Das-Gupta and Gang (2002). Limited empirical evidence is in Ebrill, *et. al*. (2001).

[4] *See* Ebrill, *et. al*. (2001).

[5] Das-Gupta and Gang (1996) compare welfare and output under revenue neutral VAT/RST and TT under fixed proportions and competitive intermediate good but not final good industries. Friedlaender (1967), and Bhatia (1992), examine price distortions.

demand curves and constant marginal costs. It also assumes substitutability between intermediate and primary inputs.

It is shown that in a "Cournot" game, where firms take each others choices of strategic variables as given,[6] a VAT which is revenue neutral to a single rate TT may not exist. Even if it does, the TT may dominate a VAT in terms of welfare, tax revenue and consumer goods output, though the converse is also possible. The same results hold in the "Stackelberg" game. In fact, if the consumer goods firm is the strategic leader, results are identical to the "Cournot" case, though this is a consequence of the assumed linear homogeneity of technology. When the intermediate input firm is the strategic leader, input prices are lower and welfare is higher than in the "Cournot" game.

The general point made in the paper is that with constraints on feasible second best taxes and the absence of competition, comparative properties of taxes are parameter dependent. The message is important only because the widely adopted VAT is shown in some cases to be inferior to taxes it has largely replaced.

a. The Framework

A profit maximising monopoly, the C-firm, produces consumer goods (Q) using an intermediate input (M) and also primary factors. The inverse demand for the consumer good is $P = f - gQ$, where f and g are positive constants. The cost function for the consumer good is: $C(Q,r)=(ar - ½br^2)Q \equiv \alpha Q$, where a and b are also positive constants and r is the cost per unit M. This is a valid cost function provided $\frac{2a}{b} > r$, being non-decreasing and concave in r.[7] From Shephard's Lemma, differentiating C with respect to r, the C-firm's derived demand for the intermediate input is $M = (a - br)Q$. The implied demand for primary factors is therefore, $C - rM \equiv C_P = ½br^2Q$. The price of primary factors is held fixed in this analysis.

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[6] Strategic variables are taken to be quantity for the consumer goods firm and price for the intermediate goods firm.

[7] If w is the composite price of other inputs, one possible cost function is with $b = 2\beta/w$, $\beta > 0$, which is linear homogenous in factor prices, as required. The implied elasticity of substitution is variable and is given by aQ/M.

A second profit maximising monopoly, the I-firm, produces the intermediate input using primary factors assumed to be available at a constant unit cost according to the function $C_I = cM$, $c > 0$. Without loss, choosing the composite primary factor as the numeraire, c is set to unity.

Proportional sales taxes at rate $t > 0$ and $s \geq 0$ are levied on the C-firm and I-firm respectively. So a VAT/RST is where $s = 0$, $t = v$. This is to be compared with a single rate TT levied at rate $s = t$. Below, the notation $T = 1\text{-}t$, $S = 1\text{-}s$, and $V = 1\text{-}v$ is used for compactness. Value-added (VA), tax revenue (G), output and welfare (W) are compared in this paper. Value added is simply the value of consumer goods, VA = PQ in this framework. Welfare is measured as the sum of consumer surplus (CS) + producer surplus + G. For G and W the expressions are

$$G = tPQ + sMr$$

$$W = CS + PQ – C_P – C_I = (½gQ^2) + PQ – (½br^2)Q - M$$

The profits of the two firms are

$$\pi_C = (fQ – gQ^2)T – (ar - ½br^2)Q \tag{1}$$

and

$$\pi_I = (a–br^2)QS – (a\text{-}br)Q. \tag{2}$$

It is easily verified that, if consumer goods and intermediate goods prices are constrained to equal marginal costs, as with competitive firms, then for any valid set of parameters and TT rate, a VAT rate can be found at which output, value added, tax revenue and welfare are higher with the VAT than with the TT.

b. *The "Cournot" Game*

The C-firm is assumed to treats input prices as given and chooses Q to maximise its profits. The I-firm chooses the input price (r), taking Q as given.

From (1) and (2) the first order conditions for profit maximisation of the two firms are (second order conditions can be seen to hold):

$$(f - 2gQ)T - (ar - \tfrac{1}{2}br^2) = 0 \tag{3}$$

and

$$[(a - 2br)S + b]Q = 0 \tag{4}$$

For future reference, note that (3) can be rearranged to get

$$Q^* = \frac{fT - (ar - \frac{1}{2}br^2)}{2gT} \equiv \frac{fT - a}{2gT} \tag{5}$$

From (3) and (4) the usual property of monopoly solutions, that optimal output is in the elastic region of demand, can be seen to hold. This immediately points to a technological restriction on the nature of strategic interaction: *If the intermediate input-output ratios are technologically given, then no equilibrium is possible when input firms take final goods output as given.* The second implication is that VA is increasing in Q.

The first order conditions are the reaction functions of the firms which must be solved simultaneously for the equilibrium values Q* and r*. Note that (4) and therefore r*, is independent of Q. The solutions are given by (6) and (7).

$$r^* = \frac{aS + b}{2bS} \tag{6}$$

$$Q^* = \frac{8bfTS^2 - (aS + b)(3aS - b)}{16bgTS^2} \tag{7}$$

Equilibrium values of M, P, G, VA and W can be found using (6) and (7). In particular, since M* = $\frac{aS - b}{2S}Q$, production will only take place if the condition $\frac{a}{b} > \frac{1}{S}$ holds, which imposes a ceiling on the feasible rate of the input tax.[8] Given this ceiling, a ceiling on the feasible rate of the output tax is implied by (7). So the analysis is valid only for these feasible rates of tax. By setting S = 1, T = V, equilibrium values of r*, Q* and M* with a VAT can be found. Similarly, setting S = T, equilibrium values with a TT can be found.

Of interest is the value of V given T, V_C, at which output neutrality obtains. Such a V can easily be shown to exist for all T such that [a/b] > [1/T]. Substituting VAT and TT tax rates into (6) and solving for V_C and T-V_C gives (8) and (9).

$$V_C = \frac{(a+b)(3a-b)T^3}{(aT+b)(3aT-b)}, \tag{8}$$

$$T - V_C = \frac{[2abT - b^2(1+T)]T(1-T)}{(aT+b)(3aT-b)}. \tag{9}$$

Given the restriction on feasible input taxation, [a/b] > [1/S], (9) implies v > t. The subscripts V and T are now used to distinguish equilibrium values of variables under the two taxes. For output neutral tax rates, the condition for higher revenues from the VAT (if such a VAT rate exists) is,

$$[1-V]PQ \geq [1-T][PQ + r_T M_T]. \tag{10}$$

This expression can be rewritten in terms of parameters as

$$\frac{[2abT - b^2(1-T)]T}{(aT+b)(3aT-b)} \; \frac{8bfTS^2 + (aT+b)(3aT-b)}{16bT^3} \geq \frac{(aT+b)(aT-b)}{4bT^2}. \tag{11}$$

Note that (11) is independent of the slope of the final goods demand curve, g. By rearranging (11) a positive lower bound for f, f_{MIN},

[8] Since $\frac{2a}{b} > r$ for positive costs but $\frac{a}{b} > r$ for M > 0, the range $\frac{2a}{b} > r \geq \frac{a}{b}$ is one case of vertical integration. Since the VAT and TT are equivalent in this framework in the absence of intermediate goods, this case is not examined further here.

can be obtained, suggesting that for a VAT to dominate a TT, demand must be sufficiently large relative to costs.[9]

c. The "Cournot" Examples

Example 1: Non-existence of a revenue-cum-output neutral VAT to a given turnover tax. The example has (a,b,f,g) = (20,0.4,2000,0.1). Furthermore, the VAT rate is chosen to be V_C, so that output is the same as that under the TT. Welfare and revenue are compared at feasible TT and VAT rates yielding non-negative profits having t = 0.004n, n = 1,2,…. and also at t = 0.0001. These examples are plotted in Figure 1.[10] Non existence of a revenue neutral VAT is illustrated by the higher revenue peak of the TT as compared to the VAT. Numerically, no revenue neutral VAT exists to the left of the Laffer curve peak for TT rates above 44.8 percent (the Laffer curve peak is at t_L, 60% < $100t_L$% < 60.03%). As can be seen, while revenue is lower under the VAT, welfare is higher.

Example 2: VAT dominated by a TT when output neutral rates exist. In Figure 2, welfare, tax revenue and output are compared for (a,b,f,g) = (20, 0.4, f_{MIN}/24, 0.1). VAT rates considered are at v =0.0012% and v = $1.2v_C$ such that TT rates are in increments of 0.04n, n = 1,2,3,…. For each of these VAT rates, existence of a revenue neutral TT rates has been verified. For example, for a VAT rate of 8 percent, the revenue neutral TT rate has 6.38% < $100t_C$% < 6.39%.

The intuition behind these results is straightforward: A part of the tax base under a TT, sales turnover of intermediate goods firms, is voluntarily given up under the VAT. With competitive intermediate input firms making zero long run profits, this does not actually result in any shrinking of the potential tax base since intermediate input firms have to shift the tax burden forward or suffer losses. Under imperfect competition or monopoly, surplus accruing to intermediate firms escapes taxation under the VAT. The examples show that this voluntary tax base

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[9] Rearrangement of (11) gives $f \geq f_{MIN} = \frac{(aT + b)(3aT - b)[4a^2T^2 - 2abT - b^2(3 - T)]}{8b^2T^3[2aT - b(1 + T)]}$.

[10] In generating data for all graphs in this paper only values for which all costs, revenues, outputs and profits are positive have been retained. Also, normalisation via linear transformations is used so that all series can be plotted on the same graph but without overlap. For G and Q, 100 and 200 respectively are added to normalised values. For W_T and W_V, the normalised variables are $100[W_i\text{-}\min(W_T,W_V)]/[\max(W_T,W_V)\text{-}\min(W_T,W_V)]$.

reduction can sometimes more than offset the production efficiency loss from the distortionary TT. One would, therefore, expect the VAT to continue to retain its superiority if final goods firms or consumers captured a greater proportion of the surplus than in the examples above, due to demand being high relative to intermediate costs. That this is indeed the case is shown in the next example.

Examples 3: TT dominated by a VAT: The parameter set for this example is (a,b,f,g) = (20, 0.4, 150000, 0.1). As can be seen the demand intercept is large relative to that in earlier examples being over 90 times as large as the largest value of f in Example 2, and over 25 times as large as the highest value of f_{MIN} for the current parameter set. Even so an exponential index had to be constructed to enable differences between the VAT and TT to be discernable in Figure 3.[11]

d. The "Stackelberg" Game

The implications of one or the other firms being a strategic leader, given the above framework, are now examined. First, as a consequence of the constant cost assumption, *when the consumer goods firm is the price leader the equilibrium is exactly the same as in the "Cournot" game*, since the **I**-firm's optimal choice of r is unaffected by the value of final goods output, Q. This, once again, illustrates the importance of technology in determining feasible market interactions.

When the **I**-firm is the strategic leader, it is shown here that (a) *the intermediate goods price is lower than in the "Cournot" game, implying higher welfare and final goods output,* and that (b*) the output neutral VAT rate to a given TT rate is higher than in the "Cournot" game. (c) The proportion of revenue going to input firms for a given TT or VAT rate is higher than in the "Cournot" game. (d) Furthermore, examples are provided of TT dominating the VAT and the converse* as in the "Cournot" game.

[11] Instead of Y_V and Y_T what is plotted is the transformed variables $X_V = 0.4 \exp(Y_V/Y_T)$ and $X_T = Y_T$ which are then normalised as described in the previous footnote.

The reaction function of the C-firm is still given by (5) (but not by (7)). However, for the I-firm (4) must now be replaced by

$$[(a - 2br)S + b]Q + \frac{p_I}{Q}\frac{\partial Q}{\partial r} = [(a - 2br) S + b]Q - \frac{(rS-1)m^2}{2gT} = 0. \qquad (12)$$

The second expression, which uses (5) and $\frac{\partial a}{\partial r} = a - br = m$, can be used to verify that the second order condition holds at the equilibrium value of r. Explicit solution of (12), which is a cubic equation in r, is discussed in the Appendix.

Comparing (12) with (4) it can be seen, as may be expected, that *the optimal choice of the intermediate input price when the I-firm is the strategic leader is below that in the "Cournot" game* (but above the competitive price) given identical parameters.[12] However, profits for the I-firm must be larger than in the "Cournot" case since the I-firm has the option of setting the same equilibrium input price but optimally chooses not to. Furthermore, from (5), the lower intermediate good price implies that final goods output is higher and its price lower in the "Stackelberg" case.

To demonstrate the second claim, set the VAT rate to V_S, the rate yielding the same output as a given TT rate. On alternately substituting VAT and TT parameter values into (5) and equating, it can be seen that

$$\frac{\alpha_V}{V_S} = \frac{\alpha_T}{T} \qquad (13)$$

Since output with no taxation is higher than that with any TT tax and since output is zero when V_S is sufficiently small, an output neutral VAT clearly exists for any given TT. The proof that $V_S < V_C$ is in the Appendix. This result suggests that less output need be sacrificed

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[12] Rearranging equation (12) gives $1/[\varepsilon_M+\varepsilon_Q]$ = [rS-1]/rS. ε_M and ε_Q are the absolute elasticities of M and Q with respect to r, and [rS-1]/rS is the percentage mark-up (or Lerner Index). In comparison, rearranging (4) yields the standard textbook condition that the inverse elasticity of demand, $1/\varepsilon_M$, is set equal to the percentage mark-up.

compared to the "Cournot" case for the VAT to dominate a given TT in revenue terms.

On the other hand the proportion of revenue, $\frac{rmQ}{PQ + rmQ} = \frac{rm}{P + rm}$ going to input firms is higher in the Stackelberg game.[13] This suggests that the TT is more likely to dominate the VAT than in the "Cournot" game. This tendency is counteracted by consumer goods output being higher in the "Stackelberg" game, so that tax base expansion and the tax rate advantage could offset the higher proportion of the tax base voluntarily given up under the VAT resulting in the VAT dominating the TT in the "Stackelberg" game. That both outcomes are possible is shown in examples below.

Nevertheless, it can be shown that revenue dominance of the TT is likely if the government's revenue needs are sufficiently high, given output neutrality. VAT revenue dominance under output neutrality requires $(T\text{-}V_S)PQ \geq (1\text{-}T)r_T M_T$. Clearly, $T - V_S > 0$ for VAT tax collection to be at least as great as TT collection implying a higher VAT tax rate than the TT rate. Since the numerators in (13) are the average costs, C_i/Q_i, i = V, T, they must be increasing in r_i, and so $r_V \leq r_T$. Substituting for V_S from (13) this condition becomes

$$(\alpha_T - \alpha_V)TP \geq (1\text{-}T)\alpha_T\left(\alpha_T - \frac{1}{2}br_T^2\right).$$

The bound on T in (14) is obtained from here.

$$1 > T \geq \frac{\alpha_T\left(\alpha_T - \frac{1}{2}br_T^2\right)}{P(\alpha_T - \alpha_V) + \alpha_T\left(\alpha_T - \frac{1}{2}br_T^2\right)}. \tag{14}$$

Since T = 1-t, (14) gives an upper bound on t in terms of the consumer goods price and average cost with a TT demonstrating the

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[13] This is easily shown by comparing $\frac{rm}{P + rm} = \frac{2i(2\alpha - ar)}{fi + \alpha + 2i(2\alpha - ar)}$, i = T, V in the "Cournot" and "Stackelberg" cases. Assuming that the ratio is at least as high in the "Cournot" case and expanding the implied inequality yields a contradiction.

likely superiority of the TT to the VAT when revenue requirements are high. Note that equations (13) and (14) do not use any special property of the "Stackelberg" game and so also apply to the "Cournot" game, though the bound will not be identical given different equilibrium values of r.

e. The "Stackelberg" Examples

Example 4: Non-existence of a revenue neutral VAT and TT dominance of the VAT The example has (a,b,f,g) = (20,0.4,700,0.1). Welfare, output and revenue are compared at feasible TT and VAT rates yielding non-negative profits having t = 0.004n, n = 1,2,… and also at t = 0.0001. VAT rate plotted are v =1.1t. These examples are plotted in Figure 4. Non existence of a revenue neutral VAT is illustrated by the higher revenue peak of the TT as compared to the VAT. Numerically, no revenue neutral VAT exists to the left of the Laffer curve peak for TT rates above 12.12 percent (the Laffer curve peak is at t_L, 16.429% < 100t_L% < 16.43%). The example shows that the TT can dominate the VAT even for realistic tax rates with price leadership.

Example 5: VAT dominance of the TT The example has (a,b,f,g) = (1.0060, 0.005, 50000, 100000). VAT rates are chosen to be v = 1.001t. The VAT dominates the TT for feasible TT rates above 2.8%. Compared to other examples in the paper, even example 3 in which the VAT dominates the TT in the "Cournot" game, the intermediate input is relatively unimportant in the production of the consumer good. Furthermore the demand for the consumer good is relatively elastic, this being the only example in which g > f. [14]

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[14] Given the unimportance of intermediate goods, welfare, output and revenue only differ by small fractions at low tax rates. In the graphs, indices of the logarithm of the percentage by which VAT values exceed TT values are plotted.

II. Concluding Comments

Given the limited relevance of price taking behaviour and fixed proportions production technologies in actual economies, particularly those of developing countries, this analysis, along with earlier negative results relating to the optimality of input taxation in the presence of constraints, cast further doubt on whether a VAT really is superior to other production and sales taxes such as the turnover tax it has often replaced. Recent empirical evidence on the output or revenue impact of the VAT in comparison with production or sales taxes it has replaced, in a major cross-country study of the VAT by Ebrill *et. al*. (2001), also does not clearly establish the superiority of the VAT.[15] Despite the special functional forms assumed, this analysis suggests that a possible partial explanation for the VAT not being clearly dominant is strategic interaction between intermediate input and final goods firms. Further empirical and theoretical comparison of the VAT with alternative taxes is required to settle the issue.

Regarding further work, first note that, in general, nothing can be said about the relation between revenue dominance of the VAT in the two models. Revenue dominance of the VAT in the "Cournot" case can, for the same parameter set and tax rates, coexist with TT dominating the VAT in the "Stackelberg" case. For example this can happen at (a,b,c,d,t,v) = (10,8,2000,1000,0.04,0.04004). The converse is also possible, for example with (a,b,c,d,t,v) = (20,0.4,700,0.1,0.036,0.0504).

However, if the cost function reflects fixed proportions production technology, say $C' = \beta rQ + \gamma Q$, then it is easily shown that the VAT always dominates the TT in revenue, output and welfare terms when the input firm is the strategic leader. So input substitutability is crucial along with non-competitive firms. Intuition and examples 3 and 5 above suggest that, given enough input substitutibility, say b exceeding a

[15] They also point to serious difficulties in discerning these effects from cross-country data, so the impact of VAT introduction is even more an open issue.

threshold value, b*, VAT dominance in the "Cournot" case should be sufficient for VAT dominance in the "Stackelberg" case. A proof of this, if indeed it is true, has so far been elusive.

The "Cournot" game can be extended to a framework with many consumer goods firms and intermediate firms with each industry being a (proper) Cournot oligopoly. Examples similar to those in this paper should continue to be possible if the number of firms is below a threshold. As is well known, the Lerner Index of monopoly power decreases as the number of firms in a Cournot oligopoly increases, tending to the competitive case in the limit. The voluntary tax sacrifice argument behind the results of this paper suggests that the results will not survive the assumption of monopolistic competition given zero profit long run equilibrium in such industries. It is not obvious how the price leadership case can be extended to many firms in the intermediate industry. Extension of the "Cournot" framework to a many sector general equilibrium model is also possible and intuition suggests that the results in this paper will survive this extension. Possibly more important, however, is extension of the framework to the context of open economies so that VAT zero rating of exports under imperfect competition can be studied.

Figure 1: Non-existence of Revenue Neutral VAT
Revenue and Welfare Indices
(VAT: Thick Lines)

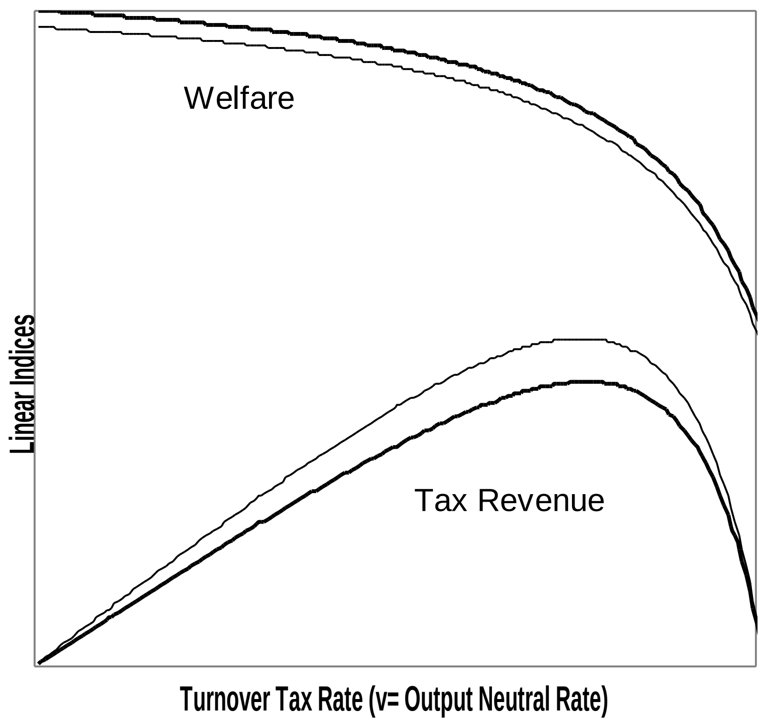


Figure 2: VAT Dominated By Turnover Tax
Output, Revenue and Welfare Indices
(VAT: Thick Lines)

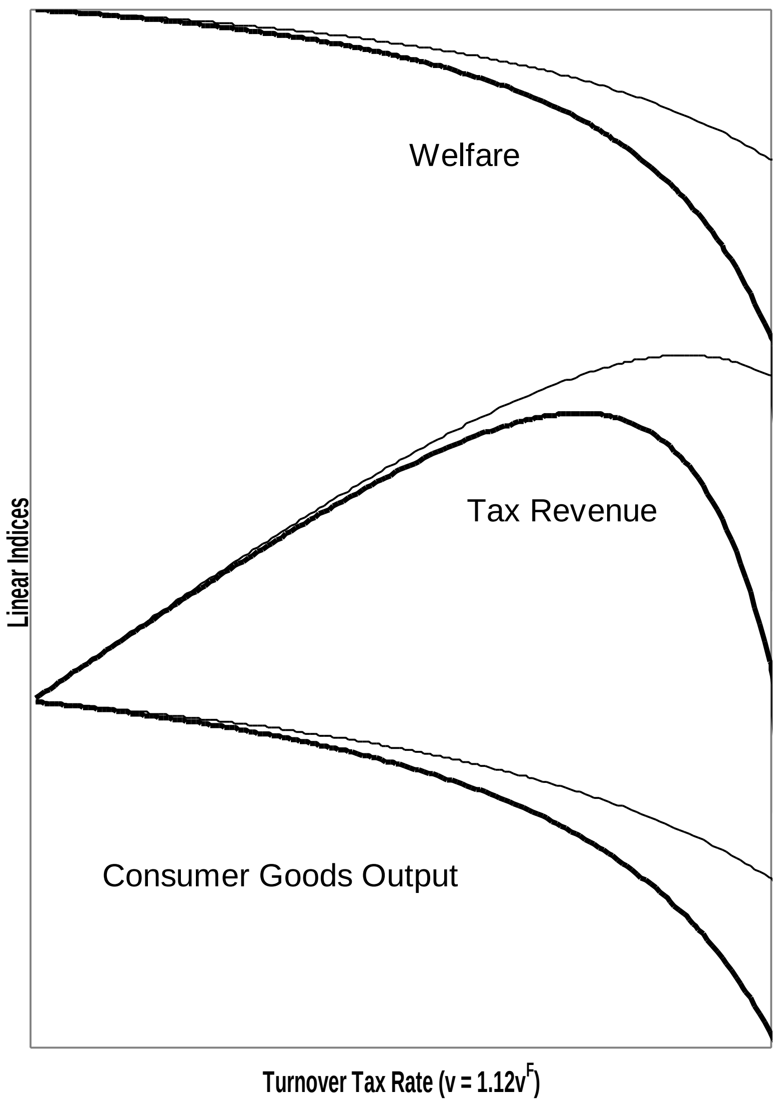
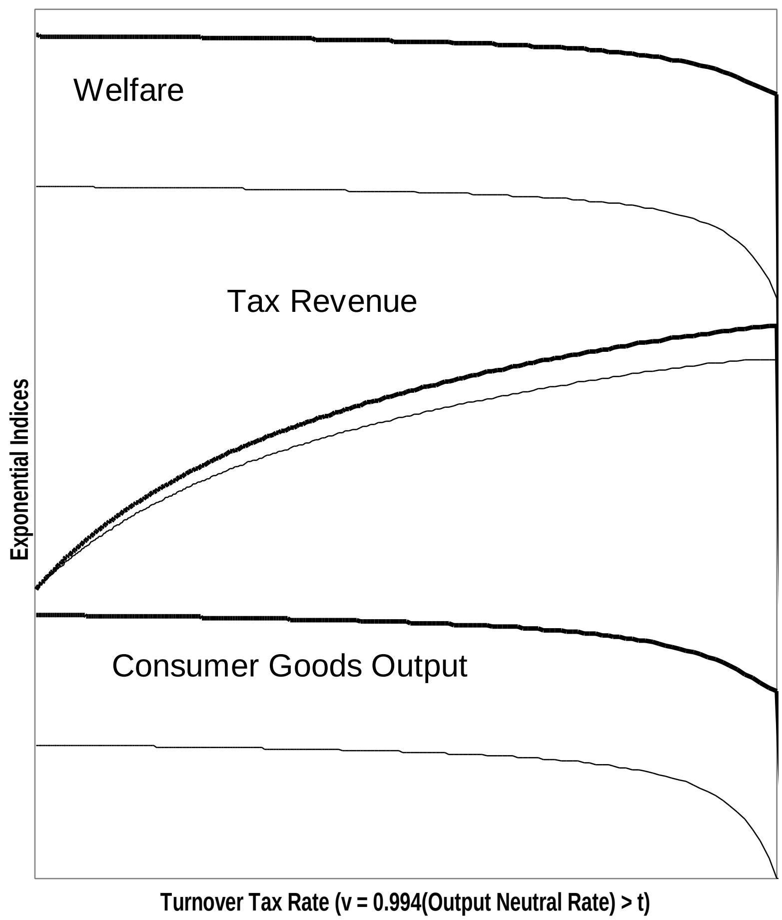


Figure 3: Turnover Tax Dominated By VAT
Output, Revenue and Welfare Indices
(VAT: Thick lines)



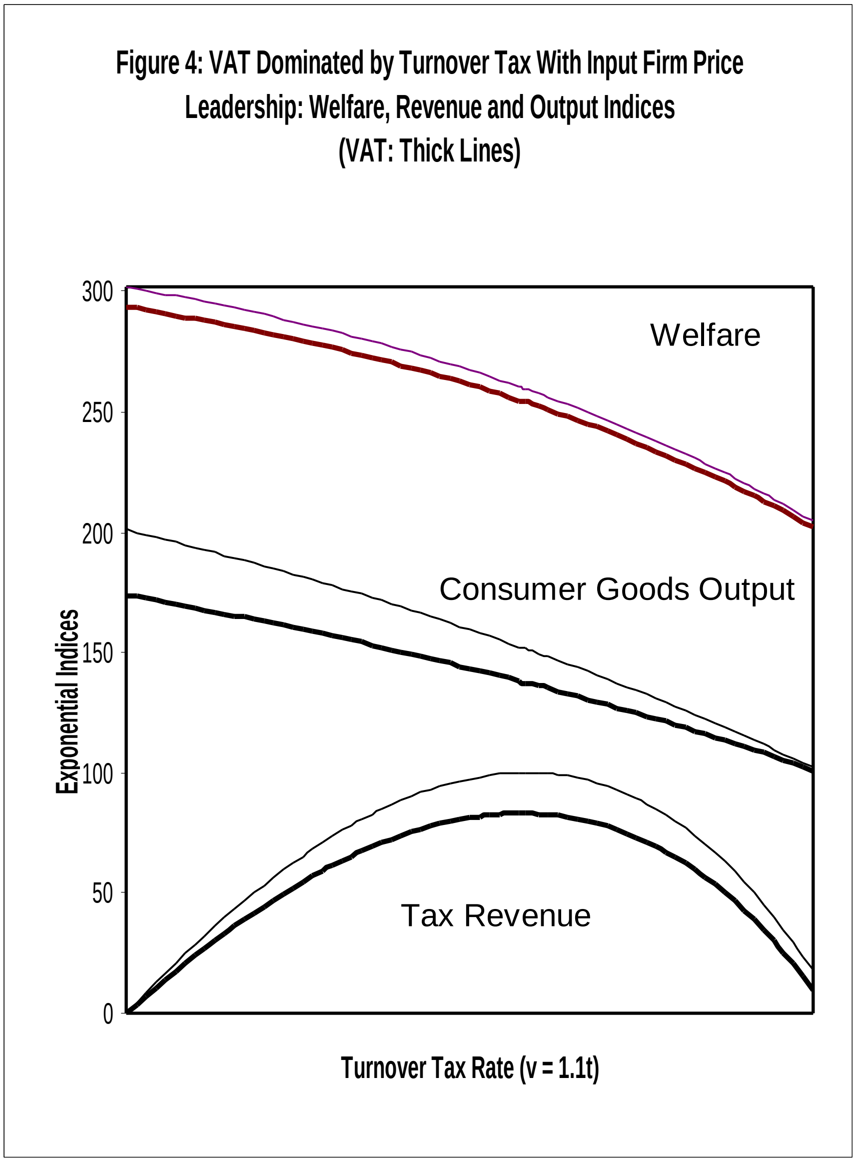
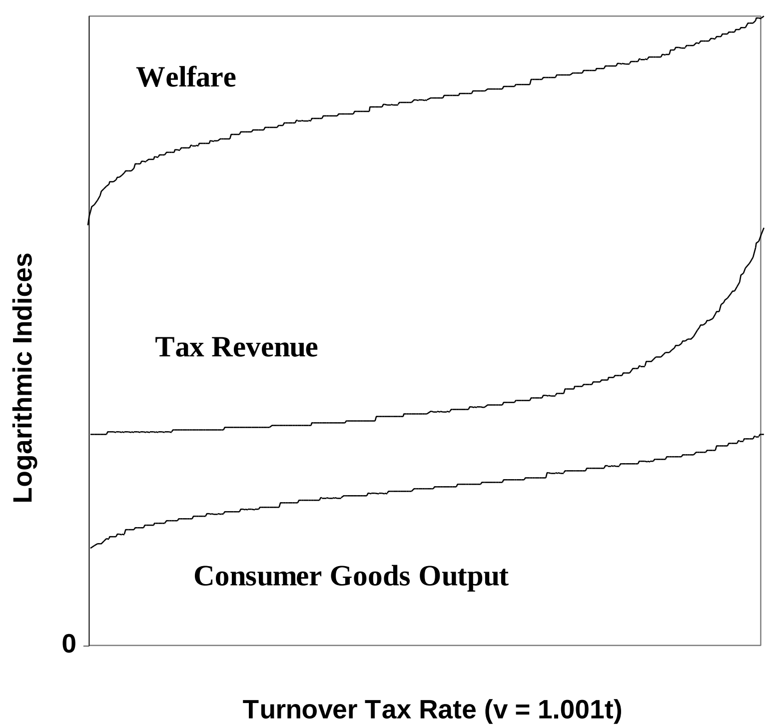


Figure 4: VAT Dominated by Turnover Tax With Input Firm Price
Leadership: Welfare, Revenue and Output Indices
(VAT: Thick Lines)
300
250
200
150
100
50
0
Exponential Indices
Welfare
Consumer Goods Output
Tax Revenue
Turnover Tax Rate (v = 1.1t)

Figure 5: VAT Dominating of TT in the Stackelberg Game: [VAT - TT]/TT



References

Bhatia, Kul B. 1992. "Indirect Taxes and Output Prices in Hierarchical Production", *Public Finance/ Finances Publiques*, 47: 347-66.

Dasgupta, Partha and Joseph Stiglitz, 1972. "On Optimal Taxation and Public Production" *Review of Economic Studies*, 39: 87-103.

Das-Gupta, Arindam and Ira N. Gang, 1996. “A Comparison of Sales Taxes”, *Public Finance/Finances Publiques*, 51: 217-225.

Das-Gupta, Arindam and Ira N. Gang, 2003. “Value Added Tax Evasion, Auditing and Transactions Matching” in (*ed*.) John McLaren, "Institutional Elements of Tax Design and Reform", World Bank Technical Paper No. 539. Washington D.C.: The World Bank.

Diamond, Peter and James Mirrlees, 1971. "Optimal Taxation and Public Production I: Production Efficiency", *American Economic Review*, 61: 8-27.

Ebrill, Liam, Michael Keen, Jean-Paul Bodin, and Victoria Summers, 2001. *The Modern VAT.* Washington D.C: International Monetary Fund.

Friedlaender, Ann F. 1967. "Indirect Taxes and Relative Prices", *Quarterly Journal of Economics*, 81: 125-39.

Newbery, David 1986. "On the Desirability of Input Taxes", *Economics Letters*, 20: 267-70.

Stiglitz Joseph and Partha Dasgupta, 1971. "Differential Taxation, Public Goods and Economic Efficiency, *Review of Economic Studies*, 38: 151-174.

Knaust, Helmuth 1998. "The "Cubic Formula", http://www.sosmath.com/ algebra/ factor/fac11/fac11.html.

Appendix

1. Proof that $V_S < V_C$

Suppose not. Then $V_S \geq V_C$, which is equivalent to $\frac{\alpha_{VS}}{\alpha_{TS}} \geq \frac{\alpha_{VC}}{\alpha_{TC}}$ or $\alpha_{VS}\alpha_{TC} \geq \alpha_{VC}\alpha_{TS.}$

To proceed further the first order condition in (12) can be used. It is convenient to express (12) in terms of m_{Cj} and m_{Sj} using the equilibrium value of $m_C = \frac{aS-b}{2S}$ from (6) and the fact that $m_{ij} = a - br_{ij}$, i=C,S, j=T, V.

Upon substitution, it can be checked that

$$r_{ij} = \frac{a-m_{ij}}{b}, \quad a_{ij} = \frac{a^2-m_{ij}^2}{2b}, \quad r_{iS}S-1 = \frac{S(2m_{iC}-m_{iS})}{b}, \text{ and}$$

$(a - 2br_{iS})S + b = 2S(m_{iS} - m_{iC})$ for i =T,V and j = C,S.

Substituting these expressions into (12) and rearranging gives

$$(m_{iS} - m_{iC})[2fib - a^2 + m_{iS}^2] + (m_{iS} - 2m_{iC})\, m_{iS}^2 = 0, \text{ i = T,V.} \qquad \text{(A1)}$$

$$\text{or } m_{iS} = \left[1 + \frac{m_{iS}^2}{2fTb - a^2 + 2m_{iS}^2}\right] m_{iC} \equiv K_i m_{iC}, \quad K_i > 1, \quad i = T,V. \qquad \text{(A2)}$$

First, note that $K_V > K_T$ when $V = V_S$.

To see this, expand K_V and K_T as given in the square bracketed expression in (A2). This shows that $K_V > K_T$ is equivalent to $\frac{2bT[f-(a_{TS}/T)]}{m_{TS}^2} > \frac{2bV[f-(a_{VS}/V)]}{m_{VS}^2}$, which simplifies, given (13), to

$\frac{m_{VS}^2}{V_S} > \frac{m_{TS}^2}{T}$. But $V_S < T$. Furthermore, $r_{TS} > r_{VS}$ under output neutrality implies that $m_{VS} > m_{TS}$.

Second, note that $m_{VC} = \frac{S(a-b)}{aS-b} m_{TC} \equiv L m_{TC}, \quad L > 1.$

Expanding $\alpha_{VS}\alpha_{TC} \geq \alpha_{VC}\alpha_{TS}$ using K_i and L gives

$$a^4 - a^2 m_{TC}^2 - a^2 m_{TC}^2 K_V^2 L^2 + m_{TC}^4 K_V^2 L^2 \geq a^4 - a^2 m_{TC}^2 L^2 - a^2 m_{TC}^2 K_T^2 + m_{TC}^4 K_T^2 L^2 \quad \text{or}$$

$$(m_{TC}^2 - a^2)(K_V^2 - K_T^2)L^2 \geq a^2 (K_T^2 - 1)(L^2 - 1).$$

Since the left hand side is negative and the right hand side is positive, a contradiction is obtained, proving the proposition.

2. Solving for equilibrium in the "Stackelberg" game

Substituting $m_{iS} = K_i m_{iC}$ in (A4) and rearranging gives

$$K_i^3 - 1.5K_i^2 + b_i K_i - B_i = 0, \quad B_i \equiv \frac{2fib - a^2}{2m_{iC}^2}, \quad i = T,V. \tag{A3}$$

This is a cubic equation in K_i which can now be solved for a real root. The method used in numerical examples is from Knaust (1998). Briefly, the method requires the substitution of $y_i = K_i - 0.5$ to be made giving rise to the equation $y_i^3 + (B_i - 0.75)Y_i - (0.25 + 0.5B_i) = 0$. Then $y_i = X_i - Z_i$ is a root of this equation where $3X_iZ_i = B_i - 0.75$, and $X_i^3 - Z_i^3 = 0.25 + B_i$. These equations in X_i and Z_i yield a quadratic in, say, Z_i^3.

Next, rearranging (A4) gives $K_i^2(K_i - 1.5) + B_i(K_i - 1) = 0$. Given $K_i > 1$, this rearrangement shows that $1 < K_i < 1.5$. In case of multiple real roots, this property permits a valid root to be selected.

Once m_{iS} is obtained, other endogenous variables are easily recovered.