# Comovement in business cycles and trade in intermediate goods

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**Abstract**

Positive correlation between intermediate goods trade and business cycle comovement raises the question of causality. Existing theories propose the direction from trade to comovement, but don't explain positive correlation of trade with TFP comovement, also in the data. My model predicts both positive correlations, and explains potential causality in the reverse direction, i.e. countries might choose trade partners based on business cycle properties. There is greater benefit in trading with positively correlated sources and self-insuring through capital accumulation, when constrained by domestic technology. I provide empirical evidence of this condition by estimating the elasticity of substitution between capital and intermediates.

JEL Codes: F4, E3, D24, C13
Keywords: Business cycle comovement, TFP comovement, intermediate goods trade

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# 1 Introduction

Intermediates in production account for an increasing percentage of world trade as countries take advantage of lower factor costs abroad. Empirical studies have highlighted that countries with such trade linkages exhibit more synchronized business cycles. This positive correlation raises the question of causality. Traditional theoretical mechanisms propose the direction where higher bilateral trade in intermediate goods causes increased business cycle correlations.[1] However, the data shows that trade is positively correlated with comovements in GDP as well as total factor productivity (TFP); and the current work in the literature explains only the first relation.[2] Based on these observations, my model makes two contributions – first, it predicts both positive correlations as seen in the data. Second, it explains potential causality in the reverse direction, i.e. countries might choose trade partners based on the properties of their business cycles.[3]

In models explaining the traditional direction of causality, a technology shock in the home country leads to an increase in domestic factor productivity and an increase in output. Since Home depends on the foreign country for intermediate goods, there is an increase in demand for the foreign good, and foreign production increases. Thus, the GDP movement is correlated. However, absent a technology shock, the foreign country is producing more output by hiring more factors, so it is not necessary that the factors are more productive. Thus in these models, an increase in trade does not lead to a positive comovement in TFP.

I address this deficiency by introducing the reverse hypothesis, where business cycle comovement affects the trading decisions of a country. In particular, the positive relation suggests that a country imports intermediate goods from countries with whose business cycles its own is synchronized. A theoretical explanation for this is a capital accumulation motive of countries. After a good technology shock, when the marginal cost of production is low, a country has incentive to import intermediate goods from a source country that has also experienced a good technology shock and has a low relative price of the good. In the presence of costs of switching trade partners countries can import during good times, build their capital and run down this capital during bad times. An alternative mechanism in which trade is caused by the properties of the business cycle is based on an insurance motive. Accordingly, countries can smooth consumption or insure against their shocks by

[1]In Burstein et al. (2008), the low elasticity of substitution between domestic and foreign intermediates in an Armington aggregate production function creates a tight dependence on the foreign good resulting in GDP correlation in response to aggregate shocks. In Arkolakis and Ramanarayanan (2009) and Zlate (2010), endogenous specialization implies that the location of production facilities is responsive to aggregate shocks and this results in the correlated movement of output.

[2] Arkolakis and Ramanarayanan (2009) and Drozd and Nosal (2008) regress the bilateral correlation of TFP on trade intensity for industrialized countries and obtain positive and significant OLS estimates.

[3]This is not to suggest that there is no causal relationship from trade to business cycles. My claim is that the reverse causality is potentially stronger in replicating the empirical features of the data.

importing from countries whose business cycles do not match their own. In this case, there is a negative relation between comovement and trade. An endowment economy will always import from a negatively comoving source to reduce the variance of consumption arising from aggregate shocks. Thus it is the presence of capital and the ability to invest that introduces the self-insurance channel.[4]

In order to get better intuition for conditions when the capital accumulation motive is stronger, I model a small open economy with two sources of imports - one that has a positive correlation and the other a negative correlation with the importing country's business cycle. The country has to choose its trade partner at the beginning of time due to high switching costs. Assuming that the importing country has full foresight about the distribution of technology shocks, it can make this choice at the beginning by comparing the expected value of its welfare with respect to each trade partner. The idea is to evaluate whether there are conditions under which the model generates higher welfare with a positively comoving trade partner and then to see if the conditions exist in the data.

The key result is that the model predicts a positive correlation between intermediate goods trade and the comovements of GDP and TFP, which is consistent with empirical evidence. This is obtained when the elasticity of substitution between capital and intermediates is low (below one). If the importing country is constrained by domestic technology and cannot easily substitute between the foreign good and domestic capital, then there is no benefit from the insurance provided by trade, and the capital building motive dominates. In the second part of the paper, I provide empirical support for the predicted complementarity by estimating the elasticity of substitution between capital and intermediates using a panel dataset.

The link between trade and cross-country business cycles has been a subject of interest in recent years in the international macroeconomic literature, especially with the proliferation of free trade agreements, currency unions and other integration initiatives. In a seminal paper, Frankel and Rose (1998) argue that joining a currency union is beneficial even if *exante* the business cycles are asynchronized, by showing empirically that countries with closer trade ties end up with highly correlated business cycles. While the basic positive relation was reiterated in a number of studies, theoretically, the impact of trade on synchronization remained unclear, and empirically, the issue of omitted variables was raised. For example, Imbs (2000, 2004) finds that similarity in sectoral specialization and financial linkages have a significant impact on cycle synchronization, whereas the impact of trade is sensitive to specifications and sub-samples. On the contrary, Baxter and Kouparitsas (2005) find that sectoral similarity does not have a robustly significant effect on output correlations, but intra-industry trade does. Inklaar et al. (2008) examine the evidence for OECD countries and find that besides trade, similarity of monetary and fiscal policies as well as specialization has a strong impact on business cycle correlations. In general, the exogeneity of the instruments used in the Frankel and

[4]The role of self insurance in incomplete markets is an established idea in the literature pioneered by Aiyagari (1994). The papers show that in order to decrease fluctuations in consumption in the presence of uninsured shocks, precautionary saving or capital accumulation is generated.

Rose empirical exercise is questioned. Theoretically, production sharing has been examined as a mechanism by which trade affects business cycle synchronization and the traditional direction of causality does not replicate the features of the data. I revisit this issue and point out that looking only at GDP comovement may not be sufficient. In order to assess the welfare that arises from integration initiatives, the models should also replicate the TFP comovement present in the data.

Thus, by suggesting potential causality in the reverse direction, i.e. from business cycles to trade, this paper adds a new dimension to macroeconomic and trade policy. While studying optimum currency areas, regional agreements and trade treaties, current and forthcoming, policy makers should be looking at the TFP correlation as an important determinant of welfare post integration.

The rest of the paper is organized as follows. Section 2 lays out the model for the small open economy with two sources of imports and provides analytical and numerical solutions for the decision problem. Section 3 presents the numerical results and intuition. Section 4 provides supporting empirical evidence of the results obtained. Section 5 concludes and proposes extensions for future work.

# 2 Model

This section develops an open economy model to obtain conditions under which a country chooses to import intermediate goods from a country with whose business cycle its own comoves positively.

A small open economy (Home) can import intermediate goods from one of two Foreign countries to benefit from the relatively low factor costs there. One foreign country has business cycles that are synchronized with Home ($H$) and the other country has business cycles that are not. I assume that there are costs of switching partners to capture the significant costs in terms of time and resources that are necessary to develop new supplier relationships or set up new production facilities. In my model, absent switching costs, swings between trading partners would occur since there is always an incentive to import from the lowest cost producer every period. In the simplest case, these switching costs are assumed to be prohibitively high, so that $H$ cannot change its trading partner once it decides to import from one of the Foreign countries. To make this optimal choice, $H$ compares its expected lifetime welfare with each trade partner.

## 2.1 Foreign Countries - Intermediate Good Producers

The time horizon is infinite and time is discrete, $t = 0, 1, \ldots$. There is an intermediate good $m$ produced by two foreign countries $F_i, i = 1, 2$. Country $i$'s efficiency in producing good $m$ at time $t$ is denoted as $z_{it}$, which is a random process. If the input cost in country $i$ is $x_{it}$, then with constant returns to scale, the cost of producing a unit of good $m$ in country $i$ is $\frac{x_{it}}{z_{it}}$. Assuming that $x_{it}$ is the same in $F_1$ and $F_2$ and normalized to one, the price in the Home country $H$ of a unit of good produced in country $i$ is

$$p_{it} = \frac{1}{z_{it}}. \tag{1}$$

Thus, the difference between the two foreign countries lies in the properties of the technology process in relation to H's technology as described below.

## 2.2 Home Country

Facing these prices for intermediate goods, a planner in $H$ equipped with initial capital stock $k_0$ chooses a sequence of future capital stocks $\{k_t\}_{t=1}^{\infty}$ and a sequence of current and future imports $\{m_t\}_{t=0}^{\infty}$ to maximize the lifetime utility of a representative household

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where $\beta \in (0, 1)$. The economy's resource constraint is

$$f(z_t k_t, m_t) \geq c_t + k_{t+1} - (1-\delta)k_t + p_t m_t, \tag{3}$$

where $\delta \in (0, 1)$ is the rate of depreciation and $c_t \geq 0$. The final good output is produced using capital and the imported intermediate good as inputs, and it is allocated to consumption, investment and the purchase of intermediate goods.[5] The absence of trade in financial assets implies that goods trade is balanced in each period.

Recall that $z_t$ and $z_{it}$ $(i = 1, 2)$ are the technology shocks associated with $H$ and $F_i$ countries respectively. If $Cov(z_t, z_{it}) > 0$ (positive comovement), then $Cov(z_t, p_{it}) < 0$, and if $Cov(z_t, z_{it}) < 0$ (negative comovement), then $Cov(z_t, p_{it}) > 0$. I set up the planning problem to obtain the value function under both cases.

Home's problem can be written as a recursive formulation of the maximization problem in terms of a Bellman equation:

$$V(k; z, p) = \max_{k', m}[u(f(k, m) - k' + (1-\delta)k - pm) + \beta V(k'; z', p')]. \tag{4}$$

Specifically,

$$V(k_t; z_t, p_t) = \max_{k_{t+1}, m_t}[u(c_t) + \beta E_t V(k_{t+1}; z_{t+1}, p_{t+1})] \tag{5}$$

such that

$$c_t = f(z_t k_t, m_t) - k_{t+1} + (1-\delta)k_t - p_t m_t, \tag{6}$$

$$c_t \geq 0. \tag{7}$$

The shocks $z$ and $p$ follow Markov random processes and are correlated positively or negatively.

[5] Labor is assumed to be inelastically supplied.

A solution to this problem is a value function that satisfies the Bellman equation and the associated policy functions mapping the current state into the optimal choice of $k$ to carry to the next period.

In the presence of very high switching costs, I solve the problem separately for the cases of positive and negative comovement. Again, since the economy chooses the optimal partner for a lifetime, and not every period, it is important to obtain the unconditional expected value, i.e., the expected value prior to the observation of the first shock. Thus $V_{pos}(k; z, p)$ ($V_{neg}(k; z, p)$) is computed as the unconditional expected lifetime utility of an agent in the home country when intermediate goods are imported from a country with synchronized (asynchronized) business cycles. This lifetime utility takes into account the distribution of positive and negative technology shocks through which the agent is going to live.

## 2.3 Solution

The problem can be solved analytically for the case of logarithmic utility in consumption, Cobb-Douglas production in capital and imports, full depreciation of capital and AR(1) shock processes. These parameters result in a knife-edge case where the covariance of the shocks does not affect the decision of the agent. In other words, the capital building motive and the insurance motive exactly offset each other. The analytical solution is presented in Appendix A.

The case specified above is restrictive and there may be plausible combinations of parameter values for which one motive may dominate the other. Intuition suggests that when the inputs are either complements or substitutes in production, the agent's expected welfare will be affected by the choice of trade partner depending on the movement of price and the nature of domestic technology. Hence, I introduce a CES production function

$$f(k_t, m_t) = [(z_t k_t)^\nu + \gamma m_t^\nu]^{\frac{1}{\nu}},$$

so as to vary the elasticity of substitution ($\frac{1}{1-\nu}$) between the inputs. The elasticity of substitution provides information about the direction and the degree of difficulty in adjusting the utilization of the inputs. The share of imports in production is determined by the parameter $\gamma$. I also relax the assumption of full capital depreciation.

The problem is solved numerically by value function iteration for a discrete set of equally spaced points for capital, $k$. I assume that the shocks $p$ and $z$ take on a range of values with some probability. For simplicity, I assume that they take three values each, say $(p_l, p_m, p_h)$ and $(z_l, z_m, z_h)$. Their evolution over time is described by the transition probability matrices. If the shocks are independent, then

$$P(p'|p) = \begin{bmatrix} \pi^*_{ll} & \pi^*_{lm} & \pi^*_{lh} \\ \pi^*_{ml} & \pi^*_{mm} & \pi^*_{mh} \\ \pi^*_{hl} & \pi^*_{hm} & \pi^*_{hh} \end{bmatrix} \text{ and } P(z'|z) = \begin{bmatrix} \pi_{ll} & \pi_{lm} & \pi_{lh} \\ \pi_{ml} & \pi_{mm} & \pi_{mh} \\ \pi_{hl} & \pi_{hm} & \pi_{hh} \end{bmatrix},$$

where for example, $\pi_{ll}$ is the probability that $z$ is in the low state at time $t$ and remains in the low state at time $t+1$.

For non-independent shocks, if the transition matrix associated with $p$ is

$$P(p'|p) = \begin{bmatrix} \pi^*_{ll} & \pi^*_{lm} & \pi^*_{lh} \\ \pi^*_{ml} & \pi^*_{mm} & \pi^*_{mh} \\ \pi^*_{hl} & \pi^*_{hm} & \pi^*_{hh} \end{bmatrix},$$

then, conditional on $p$ being in the low state, the transition matrix associated with $z$ is

$$P(z'|z) = \begin{bmatrix} \pi_{ll}+\epsilon & \pi_{lm}-\epsilon/2 & \pi_{lh}-\epsilon/2 \\ \pi_{ml}+\epsilon & \pi_{mm}-\epsilon/2 & \pi_{mh}-\epsilon/2 \\ \pi_{hl}+\epsilon & \pi_{hm}-\epsilon/2 & \pi_{hh}-\epsilon/2 \end{bmatrix},$$

where $\epsilon > 0$ implies negative comovement and $\epsilon < 0$ implies positive comovement. To understand the pair of matrices, consider the example of negative comovement. Given that $p$ is in the low state, the probability that $z$ is also in the low state or moves to the low state from the medium and high states is higher than the probability that $p$ switches to the medium or high states. Similar matrices are set up conditional on $p$ being in the medium and high states. [6]

Combining the shocks $z$ and $p$, there are nine possible states of the economy, and $V(k, zp)$ is the optimal value of the objective function, starting from the state $(k, zp)$. In order to evaluate the unconditional value function for each $k$, i.e., the welfare of the agent prior to observing the first shock, I obtain the stationary vector associated with the joint transition probability matrix and the corresponding value function.

# 3 Results

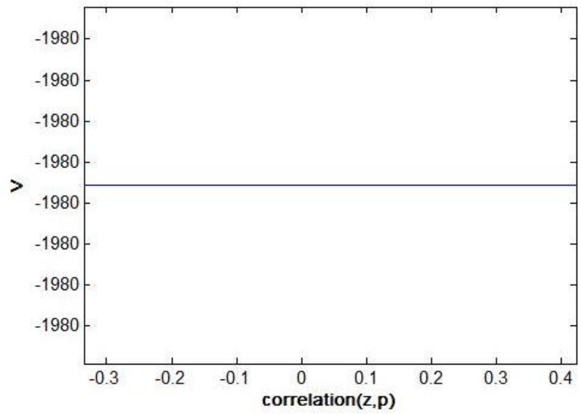
In this section, I discuss the effects of the correlation properties on the trading decision based on the numerical results of the value function iteration for specified parameters. In addition, I examine how the effects are altered by aspects of the model, namely the elasticity of substitution between domestic capital and the foreign intermediate, the depreciation rate of capital, the share of imports in production and final good imports.

Figure 1 plots the expected value function for an average value of capital against a range of values for $corr(z,p)$, for the case of log utility in consumption ($\frac{1}{1-\nu} \approx 1$), and Cobb-Douglas production function with full depreciation of capital. The parameter for the share of imports in production $\gamma$ is set at 0.15 as in Kose and Yi (2002). This is the case closest to the analytical example discussed. The graph

[6] The initial matrices for $p$ and $z$ are set up so that the relation between $\epsilon$ and $corr(z,p)$ is almost one-to-one for a simulated series of $corr(z,p)$. The entries of the probability matrix are positive and the rows sum to one, hence a nearly symmetric range of values for the $corr(z,p)$ is restricted to $[-0.3, 0.4]$. The details are in Appendix B.

**Figure 1** $\beta = 0.95, \delta = 1, \nu = 0.0001/-0.0001, \gamma = 0.15$

Plot of the expected value function against a range of values for $corr(z, p)$, for the case of log utility in consumption, and Cobb-Douglas production function with full depreciation of capital.

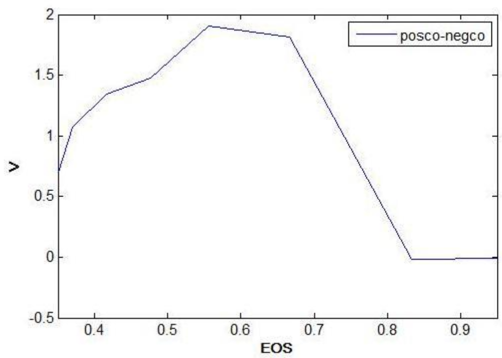


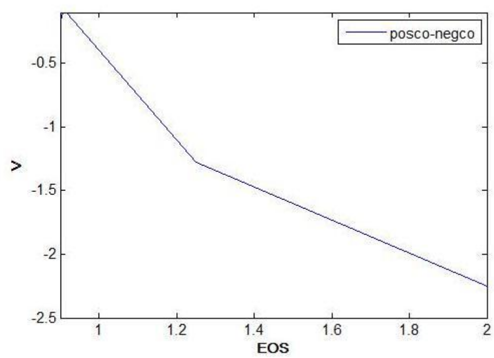

shows that the numerical solution coincides with the analytical result, i.e. the value function is not affected by the correlation of the shocks. A change in $\delta$ to a more realistic value of 0.025 does not alter this result.

## 3.1 Elasticity of substitution

**Figure 2** Difference in value function against elasticity of substitution

Plot of difference in value function under positive and negative correlation against elasticity. When the elasticity is below one (graph on the left), the value function is higher when there is positive comovement and when the elasticity is above one, the result reverses




Holding all other parameters constant, I vary the elasticity of substitution ($\sigma$) in a CES production function to deviate from the knife-edge case. Since $\sigma$ is bounded on the lower side by 0 and $\nu$ is bounded on the upper side by 1, the range of possible discrete values for $\nu$ is $[-9, 0.9]$ and for $\sigma$ is $[0.1, 10]$. Figure 2 plots the difference between the expected value functions under the cases of positive and negative comovement, i.e $V_{pos} - V_{neg}$ for average capital against a range of values for elasticity. The graphs indicate that when this parameter is different from one, the comovement of the shocks affects the value function. In particular, when the elasticity is above one, the value function is higher when there is negative comovement and when the elasticity is below one, the result reverses[7].

To gain a clearer understanding, I pick two values for $\nu$ and hence the elasticity and discuss the results for changes in the depreciation rate.

**Case 1**: $\nu = 0.3, Elasticity = 1.4$

In this case, the value of $\nu$ is 0.3, which implies that the elasticity of substitution is greater than 1 at 1.4.

**Figure 3** Value function against correlation

Plot of the expected value function for an average level of capital against a range of values for $corr(z, p)$. The graph on the left is for $\delta = 1$ and on the right is for $\delta = 0.025$. The value function is higher under negative comovement ($corr(z, p) > 0$)

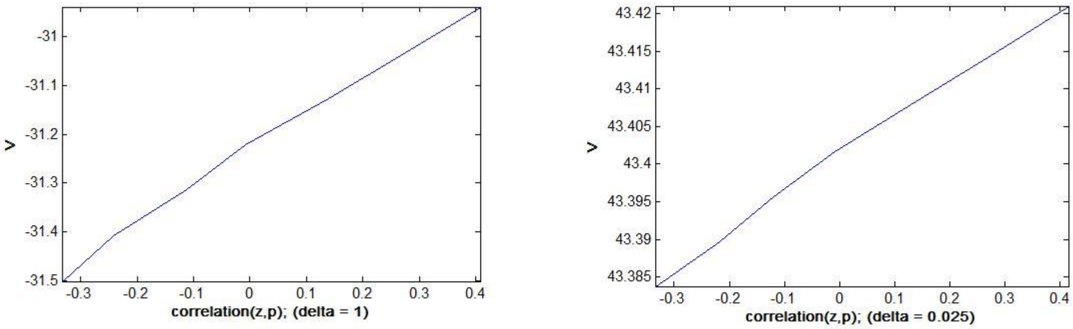


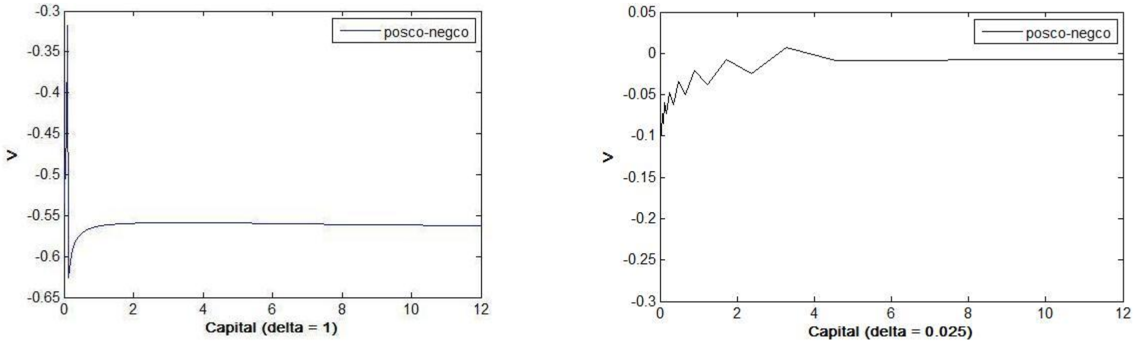

Figure 3 plots the expected value function for an average level of capital against a range of values for $corr(z, p)$, for $\delta = 1$ and $\delta = 0.025$. Recalling that $corr(z, p) < 0$ implies positive comovement, the graphs show that the value function is higher under negative comovement. In Figure 4, I plot the difference between the expected value functions under the cases of positive and negative comovement, i.e $V_{pos} - V_{neg}$, against capital for two values of $\delta$. The correlation between $z$ and $p$ is set at -0.3 and 0.3 respectively. Again, the graphs show that when the elasticity of substitution is above one, the value function is higher when a country imports from a negatively comoving source for all values of capital, so $V_{pos} < V_{neg}$.

When the elasticity of substitution between inputs is high (above 1), depending on price changes, the economy can more easily substitute one input for the other in

[7] The plot is non-linear since the shares of factors in production vary with elasticity in a CES function and $\gamma$ is fixed.

**Figure 4** Difference in value functions against capital

Plot of difference between the expected value functions under positive and negative comovement against capital for different $\delta$. When the elasticity of substitution is above one, the value function is higher when a country imports from a negatively comoving source.



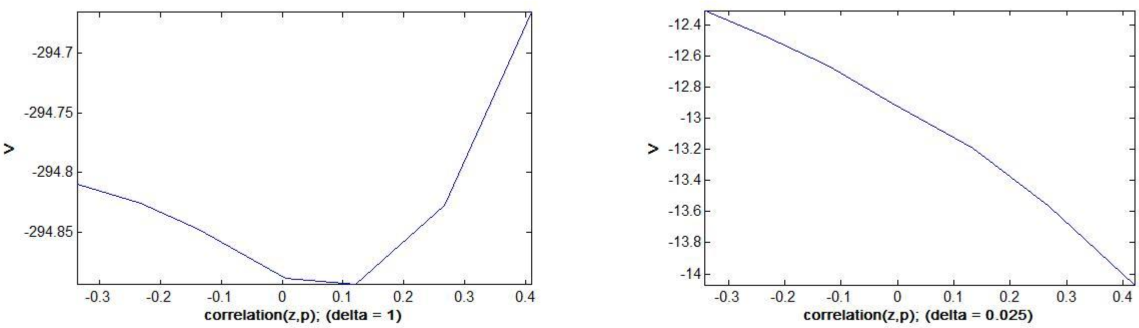

production. Here, the insurance motive dominates.

**Case 2**: $\nu = -1, Elasticity = 0.5$

Now I change the value of $\nu$ to equal $-1$, which implies that the elasticity of substitution is below 1 at 0.5. Looking at Figures 5 and 6, it is clear that for a low elasticity of substitution, in general the result is reversed, i.e., $V_{pos} > V_{neg}$ for the range of correlation values and the range of capital. Particularly, when $\delta = 0.025$, the value function is higher when there is positive comovement.

**Figure 5** Value function against correlation

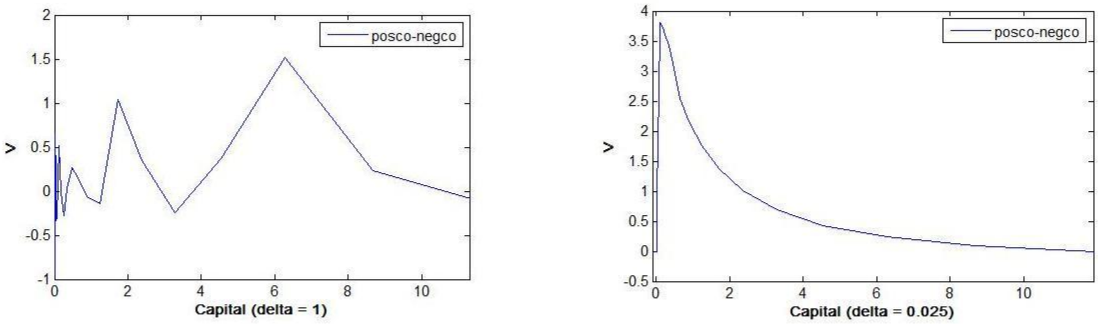
Plot of the expected value function for an average level of capital against a range of values for $corr(z, p)$. The graph on the left is for $\delta = 1$ and on the right is for $\delta = 0.025$. The value function is higher under positive comovement ($corr(z, p) < 0$)




This case shows that when the elasticity of substitution between domestic capital and imported intermediates is low (below 1), and capital depreciates slowly, the

**Figure 6** Difference in value functions against capital

Plot of difference between the expected value functions under positive and negative comovement against capital for different $\delta$. When the elasticity of substitution is below one, the value function is higher when a country imports from a positively comoving source, when $\delta = 0.025$.




capital building motive dominates the insurance motive. A low elasticity implies that when the price of the intermediate good falls, imports increase by a smaller proportion. In other words, the economy cannot use the intermediate good instead of domestic capital and is unable to benefit from the fall in price. Conversely, an increase in the price implies that the fall in imports is of a smaller proportion and hence importing from a country with opposite business cycles reduces welfare. The economy is constrained by the technology and domestic capital and there is not much benefit from insurance. The greater benefit comes from building domestic capital in good times by importing from a country with positively correlated cycle and running down this capital stock during a bad technology shock.

## 3.2 Share of imports

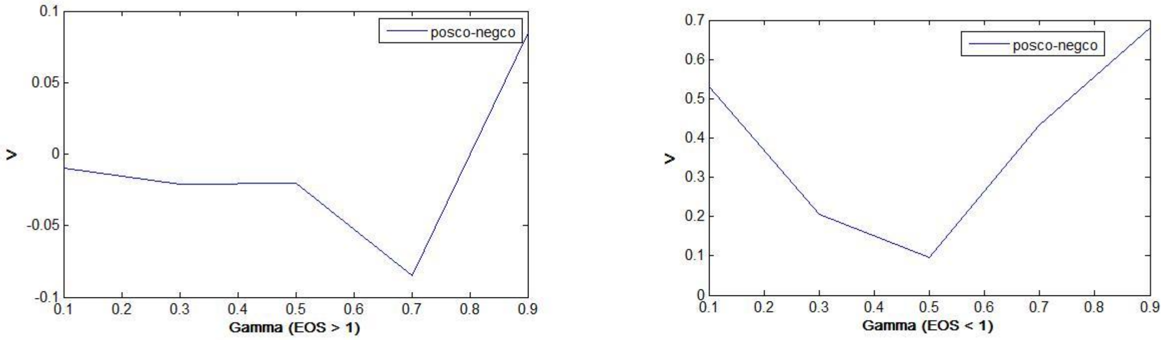
Figure 7 plots $V_{pos} - V_{neg}$ against a range of values for $\gamma$, the parameter that determines the share of imports in production. This is for correlations of 0.3 and $-0.3$ and the average value of capital. When the elasticity is above 1, the first plot shows that for a high $\gamma$ (above 0.8), $V_{pos} > V_{neg}$. This suggests that when intermediate imports contribute to a significant portion of production, even with a high elasticity of substitution, the self-insurance motive is strong. The plot on the right side shows that when the elasticity of substitution is below 1, for all values of $\gamma$, the value function is higher under positive comovement.

## 3.3 Import of final goods

The above results imply that when an economy imports intermediate goods and cannot easily substitute them with domestic capital in production in response to price changes, it is more beneficial to build capital to smooth consumption over

**Figure 7** Value function against $\gamma$

Plot of $V_{pos} - V_{neg}$ against a range of values or $\gamma$, the parameter that determines share of imports in production. When elasticity is below 1, the value function is higher under positive comovement.




its lifetime. This can be done by trading with a synchronized partner. If on the other hand, the economy imports final goods, then it can insure against consumption variance by trading with an asynchronized partner. To verify this, I modify the model so that the imported good provides utility directly and is not used in production. Thus,

$$U = \sum_{t=0}^{\infty} \beta^t (b \log c_t + (1-b) \log m_t) \tag{8}$$

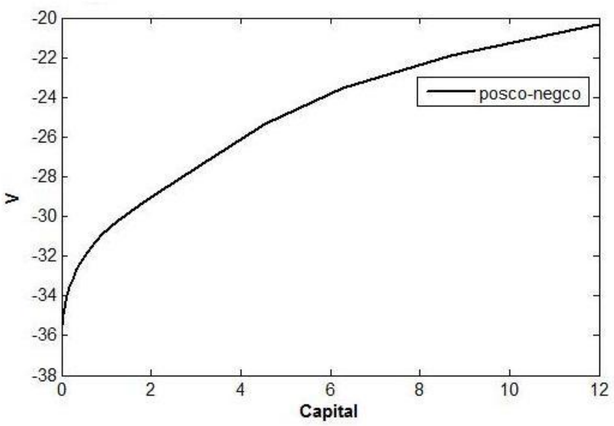
subject to

$$f(k_t) = c_t + k_{t+1} - (1-\delta)k_t + p_t m_t, \tag{9}$$

where $f(k_t) = z_t k_t^{\alpha}$ and $0 < \alpha < 1$. For parameter values $\alpha = 0.3$, $b = 0.5$

**Figure 8** Difference in value functions against capital

Plot of the difference in value function under positive and negative comovement against capital. The value function is higher under negative comovement, for final goods imports.



and $\delta = 0.025$, Figure 8 shows that the value function is higher under negative comovement for the range of capital as intuition suggests.

# 4 Empirical tests

In this section, I provide evidence about the empirical validity of the model's prediction. In particular, the parameter of importance is the elasticity of substitution between capital and intermediates and I explore the data to determine whether these inputs are complements in production in manufacturing industries. A number of empirical papers estimate elasticity parameters between inputs at the industry level. For instance, Saito (2004) estimates the Armington elasticities between intermediate imports and domestic intermediates using bilateral and multilateral data for OECD countries by industry. Gallaway et al. (2003) provide short and long run estimates for the elasticity of substitution between domestic goods and imports for over 300 manufacturing industries in the US. However, since the self insurance role played by domestic capital is key for driving the results of the model described above, it is important to obtain the elasticity parameter specifically between capital and intermediates. Hence I estimate the elasticity of substitution for OECD countries by industry using a panel dataset.[8] As a proxy, I use data on intermediate goods, which includes both domestic as well as foreign intermediates, due to the lack of data on intermediate imports at the industry level for OECD countries for a long time period.[9]

## 4.1 Estimating the EOS

Assuming the decision maker in the economy is the industry, the inputs of industry specific $(j)$ capital and intermediate goods in country $i$ enter the production function as follows:

$$Y_{ijt} = [\theta_{ij} k_{ijt}^{\frac{\sigma_{ij}-1}{\sigma_{ij}}} + (1-\theta_{ij}) m_{ijt}^{\frac{\sigma_{ij}-1}{\sigma_{ij}}}]^{\frac{\sigma_{ij}}{\sigma_{ij}-1}} \tag{10}$$

where $\sigma_{ij}$ is the time invariant elasticity of substitution between the inputs and $\theta_{ij}$ is the share of capital in production. Constrained optimization of the equation above yields the log-linear specification

$$\ln \frac{k_{ijt}}{m_{ijt}} = \sigma_{ij} \ln \frac{\theta_{ij}}{1-\theta_{ij}} + \sigma_{ij} \ln \frac{P_{ijt}^m}{P_{ijt}^k} \tag{11}$$

where $P_{ij}^m$ and $P_{ij}^k$ are the prices of intermediates and capital respectively. This equation may be stylized to fit the linear regression equation:

$$\ln y_{ijt} = \beta_{0ij} + \beta_{1ij} \ln x_{ijt} + \epsilon_{ijt} \tag{12}$$

[8] Note that the same estimation will hold if labor is included in the production function, i.e., $f(X, L)$ where $X = [\theta k^{\frac{\sigma-1}{\sigma}} + (1-\theta) m^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$

[9] Johnson and Noguera (2011) construct a global bilateral input-output table by combining input-output tables and bilateral trade data of many countries for the year 2004. This determines foreign import in each industry of the destination country by source.

where $y_{ijt}$ is the capital-intermediate goods ratio, $x_{ijt}$ is the ratio of intermediate price-capital price and $\epsilon_{ijt}$ is the independent and identically distributed error term. The elasticity of substitution between capital and intermediate imports, $\beta_{2ij}$ is the coefficient of interest. $\beta_{0ij}$ is an unobserved time invariant country-industry specific effect.

### Data

The four data series that are required to operationalize equation 4.3 are quantity of intermediate goods inputs, quantity of capital inputs, price of intermediates, and price of capital. I obtain data from the EUKLEMS database at the two-digit industry level for 16 industries (1970 - 2007) and 10 industrial countries.[10] For quantity of capital and intermediate inputs, the series used are Capital services - volume indices, and Intermediate inputs - volume indices. Intermediate inputs - price indices are the series for intermediate goods prices. All the indices use 1995 as the base year. The series for price of capital is constructed by dividing Capital compensation (in millions of local currency) by the capital volume indices. The left-hand side variable in the regression equation is constructed by dividing the volume of capital by the volume of intermediates and the right-hand side variable is obtained by dividing the price of intermediates by the price of capital.

### Estimation Procedure

The goal is to estimate the elasticity of substitution between capital and materials for each industry and country, i.e., the parameter $\sigma_{ij}$ in equation 4.2. This is done by employing panel data techniques to take advantage of the greater variability in the panel data compared to pure time series or pure cross section data, and to be able to estimate the country-industry specific parameters.

Consider the econometric specification

$$\ln y_{ijt} = \beta_{0ij} + \beta_{1ij} \ln x_{ijt} + \epsilon_{ijt}, \tag{13}$$

where $t = 1, 2, \dots T$ and $\beta_{1ij}$ is the elasticity of substitution and $\beta_{0ij}$ is the unobserved fixed effect. This is a large T panel with fixed effects, endogenous explanatory variables and an error covariance matrix that is not proportional to the identity matrix. Endogeneity could arise because of division bias, i.e., the real quantities are not independent of the prices. They are nominal values deflated by the same $P$'s that appear on the RHS and if there is measurement error in $P$, then there is endogeneity. Given the panel nature of the data, a natural choice for instruments is the lagged values of the right hand side variables. I use two lags of the log of price ratios as instruments. Since the price series is highly persistent, the lags serve as good instruments for the regressor. I report the standard errors that are robust to

[10]EU KLEMS Growth and Productivity Accounts: November 2009 Release. The countries included are Australia, Austria, Denmark, Finland, France, Italy, Nethrlands, Spain, UK and USA. The data for Austria and USA are available from 1977 - 2007.

heteroskedasticity and autocorrelation.[11]

### Results

The table below reports the estimation results for 10 countries and 16 industries. Of the 160 estimated parameters, 67 percent are positive and significant at the 10 per cent level.[12] The average elasticity estimate is 0.35, with a range between 0 and 1.66. 109 estimates are below 0.8 and only two are above one. The results are in line with Bruno (1984) who estimates the elasticity parameter as 0.3 for the manufacturing sector in ten OECD countries.[13] These results show that capital and intermediates are complements in production in most industries in the OECD countries.

[11] The covariance matrix is estimated using a heteroskedasticity and autocorrelation consistent (HAC) estimator (Newey-West).

[12] Note that the price (ratio) series in equation 11 is inverted, so the elasticity estimates are positive.

[13] Bruno (1984) estimates the elasticity of substitution between material inputs and a value added function of capital and labor for the period 1956 - 1978.

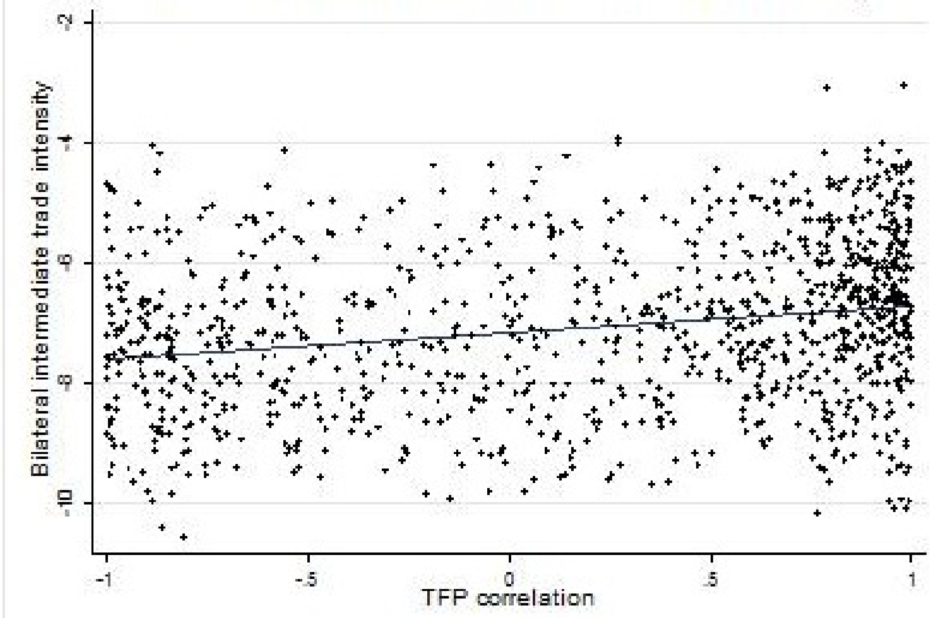
| | (1) Australia | (2) Austria | (3) Denmark | (4) Finland | (5) France | (6) Italy | (7) Netherlands | (8) Spain | (9) UK | (10) USA |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 Agriculture, Hunting, Forestry and Fishing | 0.64* | -0.11* | -0.17* | -0.06 | -0.19* | 0.47 | 0.13* | 0.41* | -0.74* | -0.19 |
| | (0.26) | (0.05) | (0.08) | (0.15) | (0.07) | (0.25) | (0.05) | (0.10) | (0.37) | (0.26) |
| 2 Mining and Quarrying | 0.14 | 0.63* | -0.11 | 0.29* | 0.47* | -0.34 | -0.20 | -0.12 | -0.08 | 0.88* |
| | (0.17) | (0.09) | (0.08) | (0.11) | (0.10) | (0.22) | (0.10) | (0.09) | (0.17) | (0.36) |
| 3 Food, Beverages and Tobacco | 0.21 | -0.57* | -0.09 | 0.91 | 0.10 | 0.21 | -0.37* | 0.67* | -0.03 | 0.04 |
| | (0.41) | (0.17) | (0.14) | (0.61) | (0.15) | (0.13) | (0.17) | (0.08) | (0.40) | (0.08) |
| 4 Textiles, Leather and Footwear | 0.43* | 0.03 | 0.03 | 0.35* | 0.23 | 0.06 | -0.05 | 0.43* | 0.12* | 0.68* |
| | (0.11) | (0.09) | (0.05) | (0.06) | (0.18) | (0.07) | (0.05) | (0.04) | (0.06) | (0.14) |
| 5 Wood and of Wood and Cork | 0.54* | 0.09 | 0.19* | 0.15* | -0.55* | 0.34* | -0.23 | 0.54* | -0.30 | 0.39* |
| | (0.13) | (0.23) | (0.09) | (0.08) | (0.26) | (0.06) | (0.12) | (0.06) | (0.15) | (0.09) |
| 6 Pulp, Paper, Printing and Publishing | 0.22 | 0.13 | 0.65* | -0.00 | 0.16 | 0.84* | 0.31* | 0.86* | -0.10 | 0.04 |
| | (0.20) | (0.32) | (0.08) | (0.09) | (0.15) | (0.12) | (0.15) | (0.19) | (0.32) | (0.27) |
| 7 Chemical, Rubber, Plastics and Fuel | -0.05 | 0.11 | 0.37* | 0.63* | -0.76* | 0.11 | 0.17 | 0.53* | 0.27* | -0.25* |
| | (0.09) | (0.09) | (0.09) | (0.28) | (0.33) | (0.15) | (0.22) | (0.15) | (0.13) | (0.11) |
| 8 Coke, Refined petroleum and Nuclear fuel | 0.08 | 0.14 | 0.22* | 0.52 | 0.20* | -0.05 | -0.34* | 0.66* | 0.14 | 0.23* |
| | (0.13) | (0.08) | (0.03) | (0.39) | (0.07) | (0.25) | (0.13) | (0.32) | (0.08) | (0.07) |
| 9 Chemicals and Chemical products | 0.17 | 0.15 | 0.46* | 0.83* | 0.95* | 0.34* | 0.75* | 0.12 | 0.13 | -0.79* |
| | (0.14) | (0.12) | (0.08) | (0.31) | (0.44) | (0.15) | (0.27) | (0.29) | (0.13) | (0.16) |
| 10 Rubber and Plastics | -0.53* | -0.03 | 0.59* | 1.07* | 0.21* | 0.58* | 0.12 | 0.65* | -0.25 | 0.36 |
| | (0.11) | (0.10) | (0.07) | (0.50) | (0.08) | (0.29) | (0.09) | (0.08) | (0.20) | (0.26) |
| 11 Other Non-metallic Mineral | -0.01 | -0.19 | 0.35* | 0.27* | -0.18 | 0.26* | -0.11 | 0.09 | 0.54* | 0.14* |
| | (0.16) | (0.29) | (0.08) | (0.10) | (0.14) | (0.07) | (0.14) | (0.39) | (0.18) | (0.04) |
| 12 Basic Metals and Fabricated Metal | 0.15 | 0.41* | 0.22* | 0.46* | 0.32 | 0.11 | -0.11 | 0.47* | -0.00 | 0.34* |
| | (0.18) | (0.12) | (0.05) | (0.08) | (0.30) | (0.08) | (0.10) | (0.06) | (0.07) | (0.08) |
| 13 Machinery, NEC | 0.09 | 0.43* | 0.47* | 0.02 | 0.01 | -0.50* | 0.00 | 0.42* | 0.16 | 0.84* |
| | (0.19) | (0.11) | (0.19) | (0.16) | (0.05) | (0.15) | (0.10) | (0.10) | (0.09) | (0.08) |
| 14 Electrical and Optical Equipment | -0.06 | 0.08 | 0.14 | 0.65* | 0.15 | 0.09 | 0.94* | 0.28* | -0.11 | 0.08 |
| | (0.10) | (0.08) | (0.11) | (0.07) | (0.30) | (0.10) | (0.21) | (0.08) | (0.08) | (0.08) |
| 15 Transport Equipment | 0.10 | 0.60* | 0.62* | 0.44* | 0.10 | 0.08* | 0.21 | 0.49* | 0.12 | 0.64 |
| | (0.07) | (0.10) | (0.10) | (0.11) | (0.11) | (0.04) | (0.12) | (0.09) | (0.08) | (0.46) |
| 16 Manufacturing NEC; Recycling | 0.09 | -0.17* | 0.62* | 0.05 | 1.66* | -0.61* | -0.06 | 0.23* | 0.22* | -0.21 |
| | (0.18) | (0.06) | (0.09) | (0.10) | (0.19) | (0.25) | (0.10) | (0.07) | (0.08) | (0.11) |

Robust standard errors in parentheses
* $p<0.05$

## 4.2 Empirical link between TFP comovement and trade

**Figure 9** Scatter plot for OECD countries

The scatter plot of TFP correlation and bilateral intermediate trade intensity shows a positive correlation.




The model predicts a positive link between TFP comovement and intermediate goods trade. To assign a magnitude to the correlation coefficient, I run the following regression:

$$log(Trade_{ij}) = \alpha_0 + \alpha_1(Corr_{ij}) + \epsilon_{ij} \tag{14}$$

where $Corr_{ij}$ is the correlation of TFP of pairs of countries. I follow the literature in defining the measure of trade intensity, $Trade_{ij}$, as the ratio of total bilateral intermediate goods trade (measured as the sum of each country's intermediate goods imports from the other) to total GDP of the two countries.

In order to take the prediction to the data, I construct a comparable measure of GDP ($Y$) from the model's output. The definition of GDP is the difference between aggregate gross output and aggregate intermediate purchases, or the sum of consumption and investment (assuming balanced trade) as below,

$$f(z_t k_t, m_t) - p_{mt} m_t = c_t + k_{t+1} - (1-\delta)k_t.$$

The data for consumption and investment are obtained from the World Bank's World Development Indicators. TFP is computed as the Solow residual from a Cobb-Douglas production function using the constructed GDP and data from the Penn World Tables.[14] The correlation of TFP for pairs of countries is obtained after HP-filtering the annual series.

[14] $TFP = \frac{GDP}{K^\alpha L^{1-\alpha}}$, $K$ denoting capital and $L$ denoting labor. The data is described in the appendix C.

The data for intermediate goods imports is taken from Comtrade, using the World Bank's classification of imports into Broad Economic Categories[15]. All the data is obtained for OECD countries for the years 1976 - 2003 and divided into five year periods.

The scatter plot in Graph 9 shows the positive relation. The OLS coefficient $\alpha_1 = 0.45$ and is significant at the 5 percent level. The interpretation of the value of the coefficient $\alpha_1$ is that a doubling of the correlation between a pair of countries results in an increase of $e^{2\alpha_1}$ of trade intensity between them. For TFP correlation, it would be 2.46.[16]

# 5 Conclusion

Intermediate goods trade accounts for an increasing proportion of world trade. Production sharing creates interdependencies across countries which makes understanding the linkages crucial for trade and macroeconomic policy. This paper takes a step in that direction by shifting attention towards an important feature of the data, namely the positive relation between TFP correlation and trade intensity, while so far the only aspect of the data that has been reproduced is the comovement of GDP correlation and trade intensity. By doing so, I raise the issue of potential causality in the reverse direction, from business cycle comovement to trade. The model predicts that countries import from positively comoving trade partners when the elasticity of substitution between capital and intermediates in low. By estimating the elasticity by industry for a panel of countries, I show that this condition exists in the data. Further, the model explains heterogeneity among countries or industries in the relation between trade and comovement i.e., when the elasticity of substitution is low, an industry imports from a positively comoving source and when the elasticity is high, from a negatively comoving source. Papers based on the original causality do not explain this heterogeneity.

An interesting implication of the model's predictions is that policy makers should be looking at the TFP correlation as an important determinant of welfare post integration.

Extensions of the model could be devoted to a fully specified characterization of the decision of the optimal trade partner in the presence of switching costs. This would allow calibration of the model to match the bilateral trade and business cycle correlation facts. The current model is sufficient however in paving the path for in depth empirical investigations regarding firstly, the direction of causality that

[15] Other measures have been used for bilateral intermediate goods trade in the literature. A number of papers, including Burstein et al. (2008) use data on intermediate trade between owners and US affiliates, reported by the Bureau of Economic Analysis for measuring production sharing. Di Giovanni and Levchenko (2010) combine sector level data on production and trade with input-output matrices to measure the extent of vertical linkages at the sectoral level.

[16] Arkolakis and Ramanarayanan (2009) regress $Corr_{ij}$ on $log(Trade_{ij})$ where the latter refers to bilateral trade intensity of final goods and obtain a positive correlation.

explains the features of the data more accurately and secondly, the role of TFP correlation in trade integration initiatives.

# A Analytical solution

The value function $V(k_t; z_t)$ must satisfy

$$V(k_t; z_t) = \max_{k_{t+1}, m_t} [\ln c_t + \beta E_t V(k_{t+1}; z_{t+1})] \tag{15}$$

such that

$$c_t = z_t k_t^{\alpha} m_t^{1-\alpha} - k_{t+1} - p_{mt} m_t, \tag{16}$$

and

$$\ln z_{t+1} = \rho \ln z_t + \epsilon_{t+1}, \tag{17}$$

where $\epsilon_t$ is an i.i.d shock.

Assume that the relation between Foreign and Home technology is

$$z_{Ft} = z_t^a$$

which implies that

$$p_{mt} = \frac{1}{z_t^a}.$$

If $a > 0$, then there is positive comovement and if $a < 0$, then there is negative comovement between the two countries' technologies.

Guess the value function,

$$V(k_t; z_t) = E + F \ln k_t + G \ln z_t. \tag{18}$$

Substituting the guess in the Bellman equation gives,

$$E + F \ln k_t + G \ln z_t = \max E_t[\ln c_t + \beta E + \beta F \ln(z_t k_t^{\alpha} m_t^{1-\alpha} - k_{t+1} - \frac{m_t}{z_t^a}) + G(\rho \ln z_t + \epsilon_{t+1})]. \tag{19}$$

The policy functions are:

$$m_t = [(1-\alpha) z_t^{1+a} k_t^{\alpha}]^{\frac{1}{\alpha}} \tag{20}$$

$$c_t = (1-\beta)\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} k_t z_t^{\frac{1+a(1-\alpha)}{\alpha}} \tag{21}$$

$$k_{t+1} = \alpha\beta(1-\alpha)^{\frac{1-\alpha}{\alpha}} k_t z_t^{\frac{1+a(1-\alpha)}{\alpha}}. \tag{22}$$

Substituting the policy functions and equating the coefficients:

$$E = \frac{1}{(1-\beta)^2}[\beta \ln \beta + (1-\beta)\ln(1-\beta) + \ln(\alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}}] \tag{23}$$

$$F = \frac{1}{1-\beta} \tag{24}$$

$$G = \frac{1+a(1-\alpha)}{\alpha(1-\rho)(1-\beta)} \tag{25}$$

The unconditional expected lifetime utility of the agent, i.e. the agent's expected welfare prior to the realization of the first shock is:

$$E_t V(k_t) = constant + \frac{1}{1-\beta} E_t \ln k_t + \frac{1 + a(1-\alpha)}{(1-\rho)(1-\beta)\alpha} E_t \ln z_t. \tag{26}$$

$E(\ln z_t) = 0$ which implies that the covariance of the shocks ($a$) does not affect the decision of the agent.

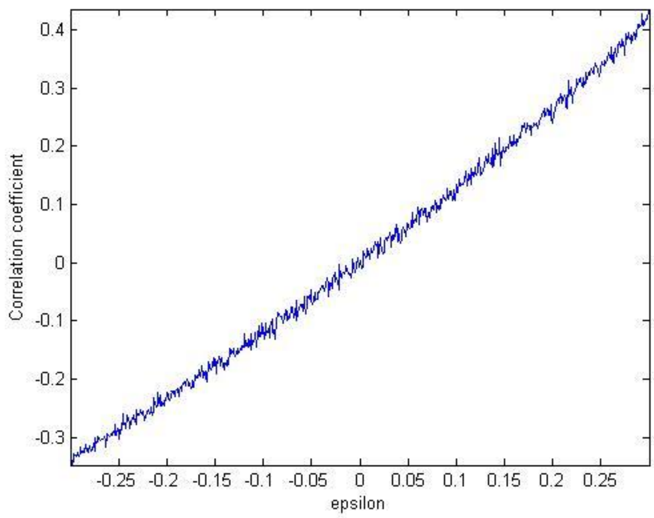
# B Simulation

I plot the simulated series of $corr(z,p)$ against $\epsilon$ for specific values in the transition probability matrices. For example, starting with

$$P(p'|p) = \begin{bmatrix} 0.9668 & 0.0332 & 0.0000 \\ 0.0109 & 0.9782 & 0.0109 \\ 0.0000 & 0.0332 & 0.9668 \end{bmatrix} \text{ and } P(z'|z) = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.3 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.4 \end{bmatrix},$$

the relation between $\epsilon$ and $corr(z,p)$ is almost one-for-one as shown in Graph 10.

**Figure 10** Relation between $\epsilon$ and simulated series for $corr(z,p)$




# C Data

1. Construction of TFP

   List of countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, UK, USA

Years: 1976 - 2003

Data source: Penn World Tables

Method: The procedure follows Caselli (2005). The series for capital is constructed using the perpetual inventory method, where $g$ is the geometric growth rate between the first available year and 1980, $\alpha$ is 0.3 and $\delta$ is 0.06 as in the literature.

2. Descriptive statistics for 4.2

**Table 1** Data for OECD countries

| | TFP correlation | Bilateral intermediate trade intensity |
|---|---|---|
| Mean | 0.21 | 0.0021 |
| Standard deviation | 0.68 | 0.0034 |
| Minimum | $-0.99$ | 0.00003 |
| Maximum | 0.99 | 0.0469 |

3. Results of the OLS regression using TFP correlation as the independent variable.

**Table 2** Regression of trade intensity on TFP correlation

| | TFP |
|---|---|
| trade intensity | 0.45 |
| | (0.04) |
| constant | $-7.15$ |
| | (0.03) |
| $R^2$ | 0.05 |

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